22206

22223

3 Hours / 70 Marks   Seat No.

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Figures to the right indicate full marks.

(4) Use of Non-programmable Electronic Pocket Calculator is permissible.

(5) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   **Solve any FIVE of the following:**   **10**

a) If $f(x) = x^2 - 2x + 5$, find $f(1) + f(2)$

b) State whether the function $f(x) = \dfrac{e^x + e^{-x}}{2}$ is even or odd, give reason.

c) Find $\dfrac{dy}{dx}$ if $y = x^2 \cdot e^x$

d) Evaluate $\int \dfrac{1}{1 + \cos 2x}\ dx$.

e) Evaluate $\int \dfrac{1}{3x + 5}\ dx$.

f) Find the area bounded by the curve $y = \sin x$, x - axis and the ordinates $x = 0$, $x = \dfrac{\pi}{2}$.

g) An unbiased coin is tossed 6 times, find the probability of getting exactly 4 heads.

Marks

2. **Solve any THREE of the following:**   12

a) Find $\frac{dy}{dx}$ if $x^2 + y^2 + xy - y = 0$ at $(1, 2)$

b) If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

c) A metal wire 36 cm long is bent to form a rectangle, find its dimensions when its area is maximum.

d) A beam is bent in the form of the curve $y = 2\sin x - \sin 2x$. Find the radius of curvature of the beam at point $x = \frac{\pi}{2}$.

3. **Solve any THREE of the following:**   12

a) Find the equation of tangent and normal to the curve $y = 4xe^x$ at origin.

b) Find $\frac{dy}{dx}$ if $y = x^x + (\sin x)^x$.

c) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{a+x}{1-ax}\right)$.

d) Evaluate $\int \frac{e^x(1+x)}{\sin^2(xe^x)} \, dx$.

4. **Solve any THREE of the following:**   12

a) Evaluate $\int \frac{1}{x^2 + 3x + 4} \, dx$.

b) Evaluate $\int \frac{dx}{3 + 2\cos x}$.

c) Evaluate $\int x \tan^{-1} x \, dx$.

d) Evaluate $\int \frac{e^x}{(e^x - 1)(e^x + 1)} \, dx$.

e) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} \, dx$.

5. **Solve any TWO of the following:** 12

a) Find the area bounded by the parabolas $y^2 = 9x$ and $x^2 = 9y$.

b) Solve the following:

i) Find the order and degree of the differential equation.

$$\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$$

ii) Solve the differential equation.

$$\frac{dy}{dx} = e^x e^{-y} + xe^{-y}$$

c) The velocity of a particle is given by $\frac{dx}{dt} = 3t^2 - 6t + 8$.

Find the distance covered in 2 seconds given that $x = 0$ at $t = 0$.

6. **Solve any TWO of the following:** 12

a) Attempt the following:

i) The probability that a person is a swimmer is 2/5. What is the probability that out of 4 persons contacted at random, exactly 1 is a swimmer.

ii) If in Poisson's distribution $P(2) = P(3)$, find $P(4)$

b) In a certain factory producing cycle tyres, there is a small change of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10. Using Poisson's distribution find the approximate number of lots containing.

i) no defective.

ii) two defective tyres respectively in a consignment of 10,000 lots.

c) A factory manufactured 2000 electric bulbs with average life 2040 hours and S.D. of 60 hours. Assuming normal distribution, find number of bulbs having life.

i) more than 2150 hours.

ii) less than 1960 hours.

Given $A(1.83) = 0.4667$

$A(1.33) = 0.4082$

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