(3 Hours)   Total Marks :80

Note: 1) Question No.1 is compulsory

2) Attempt any Three from the remaining

Q1

a) Find $L[sinh^5 t]$ **5**

b) Find a , b , c, d ,e if
$f(z)= (ax^3 +by^2x + 3x^2 +cy^2 +x)+i(d\ x^2y - 2y^3 +exy +y)$ is analytic   **5**

c) Find half range sine series of $f(x) = x(\pi - x)$ in $(0 , \pi)$ **5**

d) If $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ Find eigenvalue of Adj(A)   **5**

Q2

a) If $L[f(t)] = \dfrac{9s}{9s^2-3s+6}$ then find $L[e^t f(3t)]$ **6**

b) Find Fourier series for $f(x)= x^2$; $-\pi < x < \pi$ and $f(x+ 2\pi) = f(x)$ **6**

c) Find analytic function f(z)=u+iv in terms of z where
$u + v = e^x (\cos y + \sin y)$ **8**

Q3

a) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form y=a sin(πx/ l) from which it is released at time t=0.Show that the displacement of a point at a distance x from one end at time t is given by   **6**

$$y = a\ sin\left(\frac{\pi x}{l}\right) cos\left(\frac{\pi ct}{l}\right)$$

b) Prove that $u=x^2 -y^2 -2xy +2x -3y$ is harmonic function hence find it's harmonic conjugate function.   **6**

c) Find the Fourier series to represent $f(x) =\begin{cases} x , & 0 < x < \pi \\ 2\pi - x , & \pi < x < 2\pi \end{cases}$ **8**

in $(0 , 2\pi)$

Q4

a) Evaluate $\int_0^\infty e^{-t} \left[\frac{\cos 6t - \cos 4t}{t} \right] dt$ **6**

b) Find inverse Laplace transform of $\frac{1}{(s-2)^2(s+1)}$ **6**

c) Is the matrix $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable ? If so find the Diagonal form of A and transforming matrix of A   **8**

Q5

a) Using Cayley Hamilton Theorem find $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$

where $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ **6**

b) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$0 \le x \le 1$ subject to the condition $u(0,t) = 0, u(1,t) = 100$,

$u(x,0)=100 (x- x^2)$ & h=0.25 for one time step   **6**

c) Find the inverse Laplace transform of

   (i) $\log[(s^2 - 4)(s^2 - 9)]$

   (ii) $\frac{s}{(s-5)^2}$ **8**

Q6

a) Find the Laplace Transform of $\int_0^t u \cos h u \sin h u \, du$ **6**

b) Find the solution of $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$,   0<x<1,

u(x,0)=0, u(0,t)=0, u(1,t)=10+t, taking h=0.25, k=0.025 for $0 \le t \le 1$ where 'h' is the step length for x axis and 'k' is the step size in time direction using Bender-Schmidt method.   **6**

c) Find inverse Laplace transform of $\frac{s}{(s^2+16)^2}$ using convolution theorem   **8**

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