(3 Hours) **Total Marks : 80**

Note: (1) Question No. 1 is Compulsory.
(2) Answer any three questions from Q.2 to Q.6
(3) Figures to the right indicate full marks.

Q1. a) Find $L[te^{3t}sint]$ **5**

b) Find a , b , c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. **5**

c) Find the Fourier expansion of $f(x) = x^2, -\pi \le x \le \pi$ **5**

d) Find the eigen values of $A^2 - 5A + 4I$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ **5**

Q2. a) i) If $L\{f(t)\} = \dfrac{s}{s^2+s+4}$, find $L\{e^{-2t} f(2t)\}$ **3**

ii) Find $L(t^2 sinat)$ **3**

b) Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2 - x^2)}{12}$, where $0 < x < \pi$. **6**

c) Find analytic function *f(z)* whose imaginary part is $e^x cosy + x^3 - 3xy^2$ **8**

Q3. a) Solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method subjected to the conditions $u(0,t) = 0, u(x,0) = 0, u(1,t) = t,$ taking h =0.25, $0 < x < 1$, upto $t = 5$. **6**

b) Determine the Harmonic Conjugate of u if u + iv is analytic
$3x^2y - y^3 = u$ **6**

c) Determine the Fourier Series $f(x) = \left(\dfrac{\pi - x}{2}\right)^2$ over $[0, 2\pi]$. Hence show that $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots = \dfrac{\pi^2}{6}$ **8**

Q4. a) Evaluate the following Integral using Laplace Transforms. **6**

$$I = \int_0^\infty e^{-t}\left(\int_0^t u.cos^2 u\, du\right) dt$$

b) Determine inverse Laplace Transform of $\frac{s}{(s^2+1)(s^2+4)}$, using Convolution theorem. **6**

c) *Is the matrix* $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ diagonalizable? If so find the diagonal form of A and transforming matrix of A. **8**

Q5. a) Find the Eigen value and the eigen vector of **6**

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & 2 & 2 \end{bmatrix}$$

b) Determine the Inverse Laplace Transform of $\log\left[\frac{s^2+a^2}{(s+b)^2}\right]$ **6**

c) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,by Crank-Nicholson simplified formula, where $u(0,t) = 0, u(4,t) = 0, u(x,0) = \frac{x}{3}(16 - x^2)$, find u_{ij} , for $i = 0,1,2,3,4\ and\ j = 0,1,2$ taking $h = 1$. **8**

Q6. a) Find the Laplace Transform of $f(t) = \frac{\cos at - \cos bt}{t}$ **6**

b) A tightly stretched string with fixed end points $x = 0$,and $x = l$, in the shape defined by $y = kx(l - x)$, where k is a constant, is released from this position of rest. Find $y(x,t)$the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ **6**

c) Determine the Inverse Laplace Transform of i) $\frac{s+2}{s^2-4s+13}$ **4**

ii) $tan^{-1}(s)$ **4**
