(3 hours) **Total marks: 80**

- ***Question No.1 is compulsory.***
- *Solve **ANY THREE** questions from the **remaining five questions.***
- *The figure to the right indicates full marks.*
- *Assume suitable data wherever required, but justify the same.*

Marks

Q. 1 Solve **ANY FOUR** questions from following. (Each question carries **5 marks**) **20**

i) Explain general FEM procedure.

ii) Define shape function and enlist the properties of shape functions.

iii) Explain h-method and p-method of FEM.

iv) Describe the significance of Jacobian Matrix in co-ordinate transformation.

v) Explain the principle of minimum total potential.

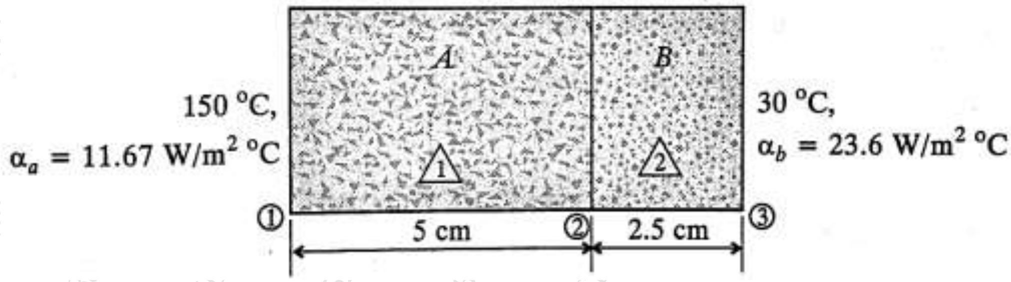
vi) Explain iso-parametric, sub-parametric and super-parametric elements.

Q. 2 a) Solve following differential equation using galerkin method **10**

$$3\frac{d^2y}{dx^2} - \frac{dy}{dx} + 8 = 0 \; ; 0 \leq x \leq 1$$

Boundary Conditions: y(0) = 1, y(1) = 2, find y(0.6)

b) Consider a plain composite wall which is made of two materials of thermal conductivity $\boldsymbol{k_a}$ = 204 W/m°C and $\boldsymbol{k_b}$ = 46 W/m°C and thickness $\boldsymbol{h_a}$ = 5 cm and $\boldsymbol{h_b}$ = 2.5 cm. Material A adjoins a hot fluid at 150°C for which heat transfer coefficient $\boldsymbol{\alpha_a}$ = 11.67 W/m^2 °C and the material B is in contact with a cold fluid at 30°C and heat transfer coefficient $\boldsymbol{\alpha_b}$ = 23.6 W/m^2 °C. Calculate rate of heat transfer through the wall and the temperature at the interface. The wall is 2 m high and 2.5 m wide. **10**



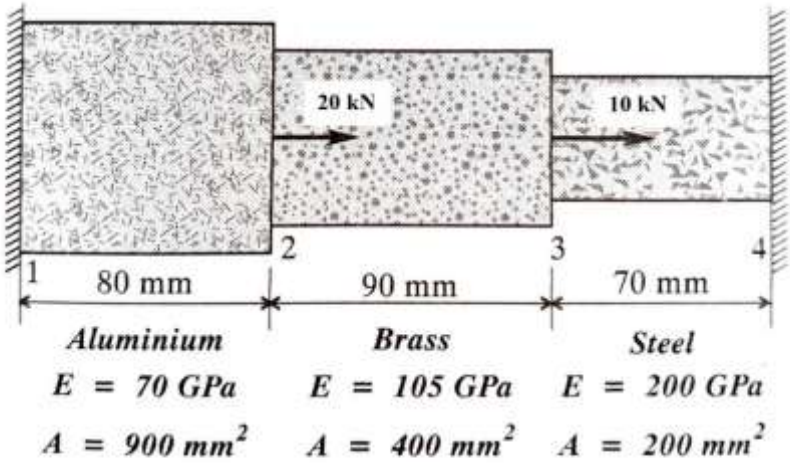
Q. 3 a) Solve the following differential equation by Rayleigh Ritz method. **10**

$$\frac{d^2y}{dx^2} - 10x^2 = 5 \;\; ; \qquad 0 \leq x \leq 1$$

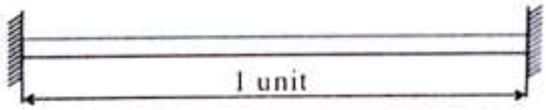
Given boundary conditions are: $y(0) = y(1) = 0$

b) Find the natural frequency of axial vibrations of a bar of uniform cross section of 50 mm^2 and length of 1 meter using consistent mass matrix. Take E = 200 GPa and density = 8000 kg/m^3. Take two linear elements. **10**

Q. 4 a) Determine the unknown reactions, displacement and element stresses for the stepped bar shown in the figure below. **10**

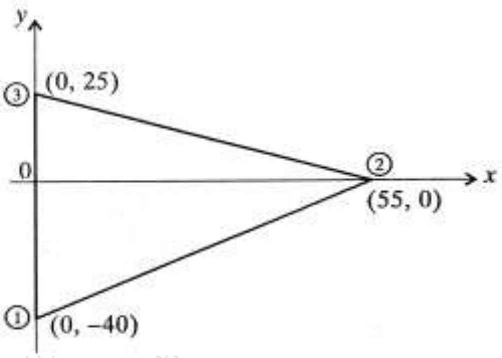


b) Determine the two natural frequencies of transverse vibration of a beam fixed at both ends as shown in figure. Use consistent mass matrix and comment on the results. Divide the whole domain into two elements of equal lengths. [Take EI = 10^6 units, $\rho A = 10^6$ units] **10**



Q. 5 a) Using the concept of serendipity, derive the shape functions for eight node rectangular element in natural co-ordinate system (ξ and η). **10**

b) The nodal displacements for the CST element shown in the figure are given as: **10**
$u_1 = 1$ mm, $u_2 = 0.5$ mm, $u_3 = 2$ mm
$v_1 = 1$ mm, $v_2 = 0.5$ mm, $v_3 = 2$ mm
Evaluate the stress for the element. Take Young's Modulus (E) = 200 GPa, Poisson's Ratio (ν) = 0.3 and thickness (t) = 1 cm.



Q. 6 a) A CST element ABC having vertices A(10, 10), B(10, 50), C(40, 10), and nodal temperatures 50 ^{0}C, 60 ^{0}C, and 80 ^{0}C respectively. Determine shape functions and nodal temperature at (20, 20). **10**

b) The following differential equation arises in connection with heat transfer in an insulated rod. **10**

$$\frac{d}{dx}\left(-K\frac{dT}{dx}\right) = q; \qquad 0 \le x \le L$$

$$\text{BCS; } T(0) = T_0 \text{ and } \left[K\frac{dT}{dx} + \beta(T - T_\infty)\right]_{x=L} = 0$$

Where T is temperature, K is thermal conductivity and q is the heat generation. Take the following data;
L= 0.1m, K=0.1 W/m^{0}C, β= 25 W/m^2 ^{0}C, $q = \bar{q} = 0$, T_0=50^{0}C and T_∞=5^{0}C.
Solve the problem using two linear finite elements for temperature values at x=L/2 and x=L. Derive the element matrix equation for the same.
